

Introduction to Abstract Algebra

Final, April 27, 2026

1. Consider strips subdivided into n blocks of size 1 or 2, where we use the color green to highlight the blocks of size 2:

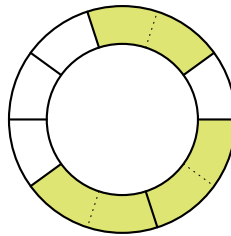


Let H_n be the number of such colorings. Find H_1 and H_2 , and express H_{n+2} in terms of H_{n+1} and H_n .

2. Fill out the following table with the first 10 values of H_n :

| | | | | | | | | | | |
|-------|---|---|---|---|---|---|---|---|---|----|
| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| H_n | | | | | | | | | | |

3. Next, consider circular bands with n equal sectors, subdivided into blocks of size 1 or 2, where we use the color green to highlight the blocks of size 2:



Find the number of such patterns that are fixed by the identity rotation, when there are n sectors. Express your answer in terms of H_n and H_{n-2} .

4. Let K_n be the number of ways to subdivide a band of n sectors into blocks of size 1 or 2 up to rotational symmetry (no reflections). Use the orbit-counting theorem to compute K_{10} .

