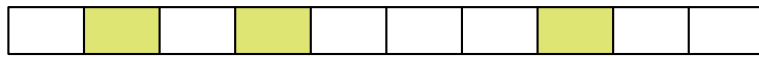


Introduction to Abstract Algebra

Practice Final, April 22, 2026

1. Consider strips subdivided into n blocks, in which some blocks are colored green, so that **no two colored blocks are adjacent**.

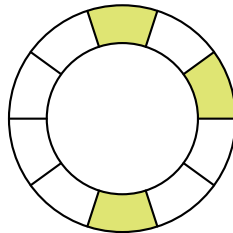


Let H_n be the number of such colorings. Find H_1 and H_2 , and express H_{n+2} in terms of H_{n+1} and H_n .

2. Fill out the following table with the first 10 values of H_n :

n	1	2	3	4	5	6	7	8	9	10
H_n										

3. Next, consider circular bands subdivided into n equal sectors, in which some sectors are colored blue, and **no two colored sectors are adjacent**.



Find the number of such patterns that are fixed by the identity element, when there are n sectors. Express your answer in terms of H_{n-1} and H_{n-3} .

4. For this exercise, recall that the Fibonacci sequence (F_n) is given by

$$F_0 := 0, \quad F_1 := 1, \quad F_{n+2} = F_{n+1} + F_n.$$

Let K_n be the number of colorings there are as described in 3 of a circular band with n sectors, up to rotational symmetry (no reflections). Use the orbit-counting theorem to prove that when p is a prime number, then

$$K_p = \frac{1}{p}(F_{p-1} + F_{p+1} + p - 1).$$

Conclude that $F_{p-1} + F_{p+1} \equiv 1 \pmod{p}$.

5. Use the orbit-counting theorem to compute K_{12} .